DEPARTMENT OF MATHEMATICS,
D.K.College, Jaleswar.
Semester-V Subject-Mathematics
Paper-Math-Maj-XII

Syllabus:
(Existence and Uniqueness of solution of IVP, Power Series solution of ordinary Differential Equations, Legendre's Differential equation and Bessel's Differential equations, Charpit's Method, Linear second order PDE,Canonical Form of 2nd order PDE and characteristics curves,One dimensional wave equation, Vibration of infinite strings, finite strings,)

UNIT-I(Existence and Uniqueness of solutions of a IVP)

1. **Short Answer Type Questions:**

(a) Define Lipschitz functions in two variables?

(b) Show that: the function $f(x,y) = x + y^2$ is bounded in the region $R : (x,y) : 0 \leq x \leq 1, 0 \leq y \leq 2$. Also find the supremum of function?

(c) Under what conditions does an IVP $\frac{dy}{dx} = f(x,y), y(x_o = y_o)$ has at least one solution?

(d) Under what conditions does an IVP $\frac{dy}{dx} = f(x,y), y(x_o = y_o)$ has unique solution?

(e) Write the sufficient condition for the function $f(x,y)$ satisfied the Lipschitz conditions

(f) Apply Picard's successive iteration method to the initial value problem $\frac{dy}{dx} = y, y(0) = 1$. Find first two iterations?

(g) Show that $f(x,y) = xy^2$ satisfies the Lipschitz condition on the rectangle $R : |x| \leq 1, |y| \leq 1$, but does not satisfy a Lipschitz condition on the strip $S : |x| \leq 1, |y| < \infty$

(h) Illustrate by an example that a continuous function may not satisfy a lipschitz condition on a rectangle?

(i) Examine the existence and uniqueness of solution of the IVP $\frac{dy}{dx} = y^{\frac{1}{3}}, y(0) = 0$

(j) Examine that which of the following functions are satisfied Lipschitz condition and also find Lipschitz constant?

(a) $\frac{dy}{dx} = 4x^2 + y^2$, on $D : |x| \leq 1, |y| \leq 1$

(b) $\frac{dy}{dx} = x^2 \cos^2 x + y \sin^2 x$, on $D : |x| \leq 1, |y| < \infty$

(c) $f(x, y) = y^{2/3}$, in the neighbourhood of origin

2. **Long Answer Type Questions**

(a) Determine the constant K(Lipschitz constant),M(Supremum of the function $f(x, y)$),and h(where h=min(a,$\frac{b}{M}$)) for the following IVP
(i) $y' = y^2, y(0) = 1 R = \{(x, y) : |x| \leq 2, |y - 2| \leq 2\}$
(ii) $y' = sinx, y(\frac{\pi}{2}) = 1 R = \{(x, y) : |x - \frac{\pi}{2}| \leq \frac{\pi}{2}, |y - 1| \leq 1\}$
(iii) (i)$y' = e^y, y(0) = 0 R = \{(x, y) : |x| \leq 3, |y| \leq 4\}$

(b) Show that the Picard's Theorem ensure a unique solution in the interval $|x| \leq \frac{1}{2}$ for the initial value problem $\frac{dy}{dx} = x + y^2, y(0) = 0$.

(c) State and prove the uniqueness theorem for the IVP $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$

(d) State and prove Picard's existence theorem for the IVP $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$

(e) Find the largest interval at which the IVP $\frac{dy}{dx} = y^2 + cos^2y, y(0) = 0$ has an unique solution in the rectangle $R : \{(x, y) : 0 \leq x \leq a, |y| \leq b, a \geq \frac{1}{2}, b > 0\}$

(f) Find the largest interval at which the IVP $\frac{dy}{dx} = y^2, y(0) = 2$ has an unique solution in the rectangle $R : \{(x, y) : 0 \leq x \leq a, |y - 2| \leq b, a \geq 0, b > 0\}$

(g) Find the largest interval at which the IVP $\frac{dy}{dx} = y^2, y(1) = -1$ has an unique solution in the rectangle $R : \{(x, y) : |x - 1| \leq a, |y + 1| \leq b\}$

(h) Consider the IVP $y' = x^2 + y^2, y(0) = 0, 0 \leq x \leq a, |y| \leq b$; Show that
:(i) The solution exists on $0 \leq x \leq min(a, \frac{b}{a^2+b^2})$
(ii) The maximum value of $\frac{b}{a^2+b^2}$ is $\frac{1}{2a}$ for a fixed a
(iii) h=min$(a, \frac{1}{2a})$ is largest when $a = \frac{1}{\sqrt{2}}$

UNIT-II(Series Solution of Differential Equations)

1. **Short Answer Type Questions**

(a) Define ordinary and singular points?

(b) Define regular singular points of a differential equation?

(c) For a differential equation $(x - 1)\frac{d^2y}{dx^2} + cot\pi x\frac{dy}{dx} + (cosec^2\pi x)y = 0$
Which of the following is true?

- (i) 0 is regular and 1 is irregular
- (ii) 1 is regular and 0 is irregular

- (iii) Both 0 and 1 is irregular
- (iv) Both 0 and 1 are regular

(d) For the differential equation $x^2(x-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$. Show that $x = 1$ is a regular singular point

(e) Write Legendre differential equation?

(f) Write standard Bessel differential equation

(g) If indicial equation of a differential equation are distinct and do not differ an integer.. Then how many Linearly independent solutions are exists

(h) Determine whether $x = 0$ is an ordinary point or regular singular point for the differential equation $2x^2y'' - xy' + (x-5)y = 0$

(i) Determine the nature of the point x=0 for the differential equation $xy'' + \sin(x)y$

(j) Prove that: $J_{\frac{-1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$

(k) Prove that: $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$

(l) Write the general solution of the following equation $x^2y'' + xy' + (x^2 - 25)y = 0$

2. **Long Answer Type Question**

(a) Find the power series solution of the equation $(x^2+1)y'' + xy' - xy = 0$ in power of x?

(b) Find the general power series solution near $x = 0$ of the legendre's equation $(1-x^2)y'' - 2xy' + n(n+1)y = 0$, where n is arbitrary constant.

(c) Find the general solution of $y'' + (x-3)y' + y = 0$ near $x = 2$.

(d) Find the power series solution of the $(x-1)$ of the IVP $xy'' + y' + 2y = 0, y(1) = 1, y'(1) = 2$ item[(e)] Show that Legendre's polynomial $P_n(x)$ satisfies the legendre's equation $(1-x^2)y'' - 2xy' + n(n+1)y = 0$,

(f) Prove that

(i) $\int_{-1}^{1} P_m(x)P_n(x)dx = 0, if m \neq m$ (ii) $\int_{-1}^{1} [P_n(x)]^2 dx = \frac{2}{2n+1}$

(g) Show that the recurrence relations :

$$\text{(i) } nP_n = (2n-1)xP_n - (n-1)P_{n-2}; n \geq 2 \quad \text{(ii) } (2n+1)P_n = P'_{n+1} - P'_{n-1}$$

(h) Prove that: $\int_{-1}^{1} xP_n(x)P_{n-1}(x)dx = \frac{2n}{4n^2-1}$?

(i) Prove that the recurrence relation: $\frac{d}{dx}(x^n J_n(x)) = x^n J_{n-1}(x)$